Online Appendix: A Marriage-Market Perspective of the College Gender Gap

Hanzhe Zhang[*]

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Abstract

The online appendix presents a model in which women are distinguished by both income and reproductive characteristics in the marriage market. Women's extra career cost k is endogenously determined rather than assumed. The main result that unambiguously strictly more women than men go to college continues to hold.

In this model, a woman does not pay a career cost k but any woman who enters the marriage market in the third period of her life is reproductively fit with probability $r < 1$, while everyone else is reproductively fit for sure. The model in which $r = 0$ captures such possibility that reproductive fitness is not observable before marriage so that all age 3 women are identically treated in their reproductive dimension. The marriage surplus a couple generates depends on the husband's income, the wife's income and her reproductive fitness. Men's marriage characteristics are H and L and women's are H, L, h, and l where lower-case letters represent lower reproductively fitness. The marriage surplus is represented by s_{HH}, s_{HL}, s_{Hh}, s_{Hl}, s_{LH}, s_{LL}, s_{Lh}, and s_{Ll}. Assume the surplus is non-negative, strictly increasing in income and reproductive fitness, strictly supermodular in incomes, $s_{HH} - s_{HL} > s_{LH} - s_{LL}$ and $s_{Hh} - s_{Hl} > s_{Lh} - s_{Ll}$, and strictly supermodular in the husband's income and the wife's reproductive fitness, $s_{HH} - s_{Hh} > s_{LH} - s_{Lh}$ and $s_{HL} - s_{Hl} > s_{LL} - s_{Ll}$.

A *stable outcome* of the marriage market $(G_m, G_w) = ((G_{mH}, G_{mL}), (G_{wH}, G_{wh}, G_{wL}, G_{wl}))$ consists of a *matching* described by $G_{\tau_m \tau_w}$ that specifies the measure of couples with a type τ_m husband and a type τ_w wife, and stable marriage payoffs $v_m = (v_{mH}, v_{mL})$ and $v_w = (v_{wH}, v_{wL}, v_{wh}, v_{wl})$ satisfying the following conditions: (1) every agent receives weakly

[*]Zhang: Department of Economics, Michigan State University, 486 West Circle Drive, 110 Marshall-Adams Hall, East Lansing, MI 48824-1038, hanzhe@msu.edu. The latest version of the online appendix is available at http://www.hanzhezhang.net/research/1707CGG_appendix.pdf.

more than being single: $v_{m\tau_m} \geq 0$ and $v_{w\tau_w} \geq 0$, (2) every matched couple divides the entire surplus: $v_{m\tau_m} + v_{w\tau_w} = s_{\tau_m\tau_w}$ if $G_{\tau_m\tau_w} > 0$, and (3) no man and no woman who are not married to each other prefer to marry each other because there is no division of surplus that would make both of them strictly better off: $v_{m\tau_m} + v_{w\tau_w} \geq s_{\tau_m\tau_w}$.

The equilibrium is defined in the same way as in the basic model. Let premiums Δu_m, Δu_w, Δv_m and Δv_w be as defined in the basic model, and define $\Delta v_{wh} = v_{wh} - v_{wL}$ and $\Delta v_{wl} = v_{wh} - v_{wL}$.

Lemma 1. *Given v_m and v_w, let $c_m \equiv p_m(\Delta u_m + \Delta v_m)$, $c_w \equiv p_w(\Delta u_w + \Delta v_m)$, and $k \equiv (1-r)[p_w \Delta v_w - p_w \Delta v_{wh} - (1-p_w)\Delta v_{wl}] > 0$. Cost $c \leq c_m$ men invest in both college and career, cost $c \leq c_w$ women invest in college, and cost $c \leq c_w - k$ women invest in college and career.*

Lemma 2. *When strategies are characterized by cutoff costs c_m, c_w, and $c_w - k$, the induced type distributions are*

$$
\begin{aligned}
G_{mH} &= F_m(c_m)p_m(2-p_m). \\
G_{wH} &= F_w(c_w)p_w + F_w(c_w-k)(1-p_w)p_w r, \\
G_{wh} &= F_w(c_w-k)(1-p_w)p_w(1-r), \\
G_{wL} &= 1 - F_w(c_w) + [F_w(c_w) - F_w(c^*_{w2})](1-p_w) + F_w(c_w-k)(1-p_w)^2 r,
\end{aligned}
$$

$G_{mL} = 1 - G_{mH}$ and $G_{wl} = 1 - G_{wH} - G_{wL} - G_{wh}$.

Lemma 3. *Stable matching is positive assortative in incomes and in husband's income and wife's fitness. Figure 1 describes the fourteen cases of stable matching.*

Lemma 4. *Table 1 describes the fourteen cases of stable marriage premiums Δv_m, Δv_w, Δv_{wh}, and Δv_{wl}.*

Theorem 1. *There exists a unique equilibrium.*

Proof of Theorem 1. Define $\Delta v_m(x) \equiv x(s_{HH} - s_{LH}) + (1-x)(s_{Hl} - s_{Ll})$ and $\Delta v_w(x)$, $\Delta v_{wh}(x)$, and $\Delta v_{wl}(x)$ associated stable marriage premiums. Define

$$
\begin{aligned}
&G_{mH}(x) \equiv F_m[p_m \Delta u_m + p_m \Delta v_m(x)]p_m(2-p_m), \\
&G_{wH}(x) \equiv F_w[p_w \Delta u_w + p_w \Delta v_w(x)]p_w + F_w[p_w \Delta u_w + r p_w \Delta v_w(x) \\
&\qquad + (1-r)p_w \Delta v_{wh}(x) + (1-r)(1-p_w)\Delta v_{wl}(x)](1-p_w)p_w r, \\
&(G_{wH} + G_{wh})(x) \equiv F_w[p_w \Delta u_w + p_w \Delta v_w(x)]p_w + F_w[p_w \Delta u_w + r p_w \Delta v_w(x)
\end{aligned}
$$

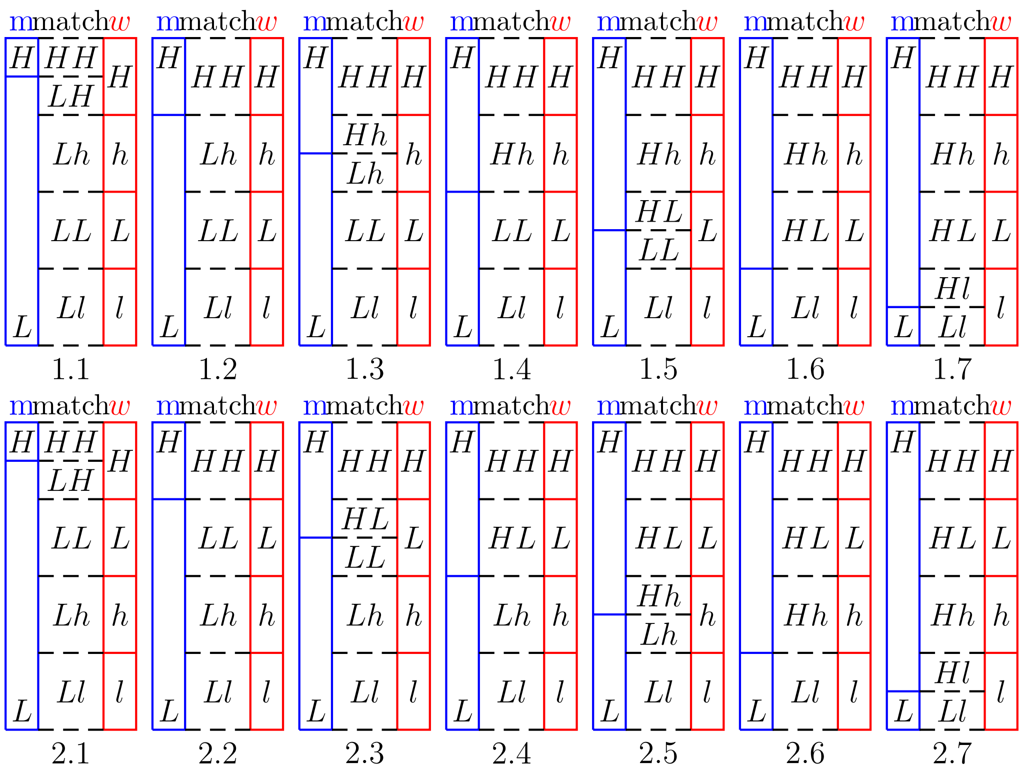

Figure 1: Stable matching.

Table 1: Stable marriage premiums.

	Δv_m	Δv_w	Δv_{wh}	Δv_{wl}
1.1	$s_{HH} - s_{LH}$	$s_{LH} - s_{LL}$	$s_{Lh} - s_{LL}$	$s_{Ll} - s_{LL}$
1.2	$(1-\lambda)(s_{HH} - s_{LH}) + \lambda(s_{Hh} - s_{Lh})$	$(1-\lambda)(s_{LH} - s_{LL}) + \lambda(s_{HH} - s_{Hh} + s_{Lh} - s_{LL})$	$s_{Lh} - s_{LL}$	$s_{Ll} - s_{LL}$
1.3	$s_{Hh} - s_{Lh}$	$s_{HH} - s_{Hh} + s_{Lh} - s_{LL}$	$s_{Lh} - s_{LL}$	$s_{Ll} - s_{LL}$
1.4	$(1-\lambda)(s_{Hh} - s_{Lh}) + \lambda(s_{HL} - s_{LL})$	$(1-\lambda)(s_{HH} - s_{Hh} + s_{Lh} - s_{LL}) + \lambda(s_{HH} - s_{HL})$	$(1-\lambda)(s_{Lh} - s_{LL}) + \lambda(s_{Hh} - s_{HL})$	$s_{Ll} - s_{LL}$
1.5	$s_{HL} - s_{LL}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL}$	$s_{Ll} - s_{LL}$
1.6	$(1-\lambda)(s_{HL} - s_{LL}) + \lambda(s_{Hl} - s_{Ll})$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL}$	$(1-\lambda)(s_{Ll} - s_{LL}) + \lambda(s_{Hl} - s_{HL})$
1.7	$s_{Hl} - s_{Ll}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL}$	$s_{Hl} - s_{HL}$
2.1	$s_{HH} - s_{LH}$	$s_{LH} - s_{LL}$	$s_{Lh} - s_{LL}$	$s_{Ll} - s_{LL}$
2.2	$(1-\lambda)(s_{HH} - s_{LH}) + \lambda(s_{HL} - s_{LL})$	$(1-\lambda)(s_{LH} - s_{LL}) + \lambda(s_{HH} - s_{HL})$	$s_{Lh} - s_{LL}$	$s_{Ll} - s_{LL}$
2.3	$s_{HL} - s_{LL}$	$s_{HH} - s_{HL}$	$s_{Lh} - s_{LL}$	$s_{Ll} - s_{LL}$
2.4	$(1-\lambda)(s_{HL} - s_{LL}) + \lambda(s_{Hh} - s_{Lh})$	$s_{HH} - s_{HL}$	$(1-\lambda)(s_{Lh} - s_{LL}) + \lambda(s_{Hh} - s_{HL})$	$(1-\lambda)(s_{Ll} - s_{LL}) + \lambda(s_{Ll} - s_{Lh} + s_{Hh} - s_{HL})$
2.5	$s_{Hh} - s_{Lh}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL}$	$s_{Ll} - s_{Lh} + s_{Hh} - s_{HL}$
2.6	$(1-\lambda)(s_{Hh} - s_{Lh}) + \lambda(s_{Hl} - s_{Ll})$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL}$	$(1-\lambda)(s_{Ll} - s_{Lh} + s_{Hh} - s_{HL}) + \lambda(s_{Hl} - s_{HL})$
2.7	$s_{Hl} - s_{Ll}$	$s_{HH} - s_{HL}$	$s_{Hh} - s_{HL}$	$s_{Hl} - s_{HL}$

Table 2: Changes as x increases: $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$.

	$s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$	
	$x \leq x_{1.5}$	$x \geq x_{1.5}$
$\Delta v_m(x)$	↓	↓
$\Delta v_w(x)$	↑	-
$\Delta v_{wh}(x)$	↑	-
$\Delta v_{wl}(x)$	-	↓
$G_{mH}(x)$	↓	↓
$G_{wH}(x)$	↑	↓
$G_{wH}(x) + G_{wh}(x)$	↑	↓
$(G_{wH} + G_{wh} + G_{wL})(x)$	↓	↑

$$
\begin{aligned}
&+(1-r)p_w\Delta v_{wh}(x) + (1-r)(1-p_w)\Delta v_{wl}(x)](1-p_w)p_w, \\
&(G_{wH} + G_{wL})(x) \equiv 1 - F_w[p_w\Delta u_w + rp_w\Delta v_w(x) \\
&\quad +(1-r)p_w\Delta v_{wh}(x) + (1-r)(1-p_w)\Delta v_{wl}(x)](1-p_w)(1-r), \\
&(G_{wH} + G_{wh} + G_{wL})(x) \equiv 1 - F_w[p_w\Delta u_w + rp_w\Delta v_w(x) \\
&\quad +(1-r)p_w\Delta v_{wh}(x) + (1-r)(1-p_w)\Delta v_{wl}(x)](1-p_w)^2(1-r).
\end{aligned}
$$

Case 1. $s_{Hh} + s_{LL} > s_{HL} + s_{Lh}$. Define

$$
\phi_1(x) \equiv \begin{cases}
0, & G_{mH}(x) < G_{wH}(x) \\
[0, x_{1.3}], & G_{mH}(x) = G_{wH}(x) \\
x_{1.3}, & G_{wH}(x) < G_{mH}(x) < G_{wH}(x) + G_{wh}(x) \\
[x_{1.3}, x_{1.5}], & G_{mH}(x) = G_{wH}(x) + G_{wh}(x) \\
x_{1.5}, & (G_{wH} + G_{wh})(x) < G_{mH}(x) < (G_{wH} + G_{wh} + G_{wL})(x) \\
[x_{1.5}, 1], & G_{mH}(x) = G_{wH}(x) + G_{wh}(x) + G_{wL}(x) \\
1, & G_{mH}(x) > G_{wH}(x) + G_{wh}(x) + G_{wL}(x)
\end{cases}
$$

where indices $x_{1.3}$ and $x_{1.5}$ represent the stable outcome in Markets 1.3 and 1.5. ϕ_1 is upper-hemicontinuous and $\phi_1(x)$ is convex for all $x \in [0, 1]$. Therefore, ϕ_1 has a fixed point, and an equilibrium exists. Equilibrium uniqueness is shown as follows.

Case 1a. $G_{mH}(x_{1.5}) < (G_{wH} + G_{wh})(x_{1.5}) < (G_{wH} + G_{wh} + G_{wL})(x_{1.5})$**.**

$\phi_1(x_{1.5}) \subseteq [0, x_{1.3}]$. For any $x < x_{1.5}$, $G_{mH}(x)$ decreases, and $(G_{wH} + G_{wh})(x)$ and $G_{wH}(x)$ increase, so $\phi_1(x)$ decreases, and there is a unique fixed point $x^* \leq x_{1.5}$. For any $x > x_{1.5}$, $G_{mH}(x) < G_{mH}(x_{1.5}) < (G_{wH} + G_{wh} + G_{wl})(x_{1.5}) < (G_{wH} + G_{wh} + G_{wl})(x)$, so $x' \in \phi_1(x)$

Table 3: Changes as x increases: $s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$.

	$s_{Hh} + s_{LL} < s_{HL} + s_{Lh}$		
	$x \leq x_{2.3}$	$x_{2.3} \leq x \leq x_{2.5}$	$x \geq x_{2.5}$
$\Delta v_m(x)$	↓	↓	↓
$\Delta v_w(x)$	↑	-	-
$\Delta v_w(x)$	-	↓	-
$\Delta v_{wl}(x)$	-	↓	↓
$G_{mH}(x)$	↓	↓	↓
$G_{wH}(x)$	↑	↓	↓
$G_{wH}(x) + G_{wL}(x)$	↓	↑	↑
$(G_{wH} + G_{wh} + G_{wL})(x)$	↓	↑	↑

implies $x \leq x_{1.5}$. Hence there is no fixed point $x^* > x_{1.5}$.

Case 1b. $G_{wH} + G_{wh})(x_{1.5}) \leq G_{mH}(x_{1.5}) \leq (G_{wH} + G_{wh} + G_{wL})(x_{1.5})$**.**

$x^* = x_{1.5}$ is a fixed point. For any $x < x_{1.5}$, $G_{mH}(x) \geq G_{mH}(x_{1.5}) \geq (G_{wH} + G_{wh})(x_{1.5}) \geq (G_{wH} + G_{wh})(x)$,so $x' \in \phi_1(x)$ implies $x' \geq x_{1.5}$. For any $x > x_{1.5}$, $G_{mH}(x) \leq G_{mH}(x_{1.5}) \leq (G_{wH} + G_{wh} + G_{wL}(x_{1.5}) \leq (G_{wH} + G_{wh} + G_{wL})(x)$, so $x' \in \phi_1(x)$ implies $x' \leq x_{1.5}$.

Case 1c. $G_{mH}(x_{1.5}) > (G_{wH} + G_{wh} + G_{wL})(x_{1.5}) > (G_{wH} + G_{wh})(x_{1.5})$**.**

$\phi_1(x_{1.5}) = 1$. For any $x \leq x_{1.5}$, $G_{mH}(x) \geq G_{mH}(x_{1.5}) > (G_{wH}+G_{wh})(x_{1.5}) \geq (G_{wH}+G_{wh})(x)$, so $\phi_1(x) = 1$. For any $x > x_{1.5}$, $G_{mH}(x)$ strictly decreases and $(G_{mH}+G_{wh}+G_{wL})(x)$ strictly increases, so $\phi_1(x)$ weakly decreases and there's a unique fixed point $x^* \geq x_{1.5}$.

Case 2. $s_{Hh} + s_{LL} \leq s_{HL} + s_{Lh}$. Define

$$\phi_2(x) \equiv \begin{cases} 0, & G_{mH}(x) < G_{wH}(x) \\ [0, x_{2.3}], & G_{mH}(x) = G_{wH}(x) \\ x_{2.3}, & G_{wH}(x) < G_{mH}(x) < G_{wH}(x) + G_{wL}(x) \\ [x_{2.3}, x_{2.5}], & G_{mH}(x) = G_{wH}(x) + G_{wL}(x) \\ x_{2.5}, & (G_{wH} + G_{wL})(x) < G_{mH}(x) < (G_{wH} + G_{wL} + G_{wh})(x) \\ [x_{2.5}, 1], & G_{mH}(x) = G_{wH}(x) + G_{wL}(x) + G_{wh}(x) \\ 1, & G_{mH}(x) > G_{wH}(x) + G_{wL}(x) + G_{wh}(x) \end{cases}$$

where indices $x_{2.3}$ and $x_{2.5}$ represent the stable outcome in Markets 2.3 and 2.5. ϕ_2 is upper-hemicontinuous and $\phi_2(x)$ is convex for all x. Therefore, ϕ_2 has a fixed point, and an equilibrium exists. Equilibrium uniqueness is shown as follows.

Case 2a. $G_{mH}(x_{2.3}) < G_{wH}(x_{2.3}) < (G_{wH} + G_{wL})(x_{2.3})$**.**

$\phi_2(x_{2.3}) = 0$. For any $x \leq x_{2.3}$, $G_{mH}(x)$ decreases and $G_{wH}(x)$ increases, so $\phi_2(x)$ decreases and there is a unique fixed point $x^* \leq x_{2.3}$. For any $x > x_{2.3}$, $G_{mH}(x) \leq G_{mH}(x_{2.3}) < (G_{wH} + G_{wL})(x_{2.3}) \leq (G_{wH} + G_{wL})(x)$, so $x' \in \phi_2(x)$ implies $x' \leq x_{2.3}$.

Case 2b. $G_{wH}(x_{2.3}) \leq G_{mH}(x_{2.3}) \leq (G_{wH} + G_{wL})(x_{2.3})$**.**

$x^* = x_{2.3}$ is a fixed point. For any $x < x_{2.3}$, $G_{wH}(x) \leq G_{wH}(x_{2.3}) \leq G_{mH}(x_{2.3}) \leq G_{mH}(x)$, so $x' \in \phi_2(x)$ implies $x' \geq x_{2.3}$. For any $x > x_{2.3}$, $G_{mH}(x) \leq G_{mH}(x_{2.3}) \leq (G_{wH} + G_{wL})(x_{2.3}) \leq (G_{wH} + G_{wL})(x)$, so $x' \in \phi_2(x)$ implies $x' \leq x_{2.3}$. Hence, $x^* = x_{2.3}$ is the unique fixed point.

Case 2c. $G_{wH}(x_{2.3}) < (G_{wH} + G_{wL})(x_{2.3}) < G_{mH}(x_{2.3})$**.**

$\phi_2(x_{2.3}) \subseteq [x_{2.5}, 1]$. For any $x < x_{2.3}$, $G_{wH}(x) \leq G_{wH}(x_{2.3}) \leq G_{mH}(x_{2.3}) \leq G_{mH}(x)$, $x' \in \phi_2(x)$ implies $x' \geq x_{2.3}$. For any $x \geq x_{2.3}$, $(G_{wH} + G_{wL})(x)$ and $(G_{wH} + G_{wL} + G_{wh})(x)$ increase and $G_{mH}(x)$ decreases in x, so $\phi_2(x)$ decreases, and there's a unique fixed point $x^* \geq x_{2.3}$. □

Proposition 1. *Suppose the setting is gender-symmetric except for reproductive fitness ($F_m = F_w$, $p_m = p_w$, $\Delta u_m = \Delta u_w$, $s_{HL} = s_{LH}$, but $r < 1$). Strictly more women than men go to college and weakly fewer fit women than men earn a high income.*

Proof of Proposition 1. Let $F_m = F_w \equiv F$, $p_m = p_w \equiv p$, $\Delta u_m = \Delta u_w \equiv \Delta u$, $s_{HL} = s_{LH}$, and $r < 1$.

Suppose by contradiction weakly fewer women than men go to college: $F(c_m^*) \geq F(c_w^*)$. It directly follows from $c_m^* = p\Delta u + p\Delta v_m^* \geq c_w^* = p\Delta u + p\Delta v_w^*$ that $\Delta v_m^* \geq \Delta v_w^*$. However, the mass of high-income men, $G_{mH}^* = F(c_m^*)p(2-p)$, is strictly more than the mass of high-income women, $G_{wH}^* + G_{wh}^* < F(c_w^*)p(2-p)$. As a result, there is always a positive mass of (H, L) couples, and they divide up their surplus: $v_{mH}^* + v_{wL}^* = s_{HL}$. Similarly, there is always a positive mass of (H, H) couples: $v_{mH}^* + v_{wH}^* = s_{HH}$. Subtracting the two conditions, we get $v_{wH}^* - v_{wL}^* = s_{HH} - s_{HL}$. Furthermore, because $v_{mL}^* + v_{wL}^* \geq s_{LL}$, $v_{mH}^* - v_{mL}^* \leq s_{HL} - s_{LL} = s_{LH} - s_{LL}$, where the equality follows from $s_{HL} = s_{LH}$. $\Delta v_m^* = v_{mH}^* - v_{mL}^* \leq s_{LH} - s_{LL} < s_{HH} - s_{HL} = v_{wH}^* - v_{wL}^* = \Delta v_w^*$, contradicting the previous conclusion that $\Delta v_m^* \geq \Delta v_w^*$.

Suppose by contradiction strictly more fit high-income women than high-income men earn a high income: $G_{mH} < G_{wH}$. Therefore, the marriage-type distributions are either of Case 1.1 or Case 2.1. In those cases, the stable marriage payoff differences are $v_{mH}^* - v_{mL}^* = s_{HH} - s_{LH}$ and $v_{wH}^* - v_{wL}^* = s_{LH} - s_{LL}$. Since $s_{HL} = s_{LH}$, $v_{mH}^* - v_{mL}^* = s_{HH} - s_{LH} = s_{HH} - s_{HL} > s_{LH} - s_{LL} = v_{wH}^* - v_{wL}^*$. However, if $v_{mH}^* - v_{mL}^* > v_{wH}^* - v_{wL}^*$, then there should be strictly more high-income men than high-income women, as $G_{mH} = F(p_m\Delta u + p\Delta v_m^*)p(2-p) > F(p_m\Delta u + p\Delta v_w^*)[p + p(1-p)] > F(p_m\Delta u + p\Delta v_w^*)p + F(p_m\Delta u + p\Delta v_w^* - k^*)p(1-p) = G_{wH}$, where k^* is the endogenous career cost. The conclusion that $G_{mH} > G_{wH}$ contradicts $G_{mH} < G_{wH}$, so we must have $G_{mH} \geq G_{wH}$. □